DOCUMENT RESUME

ED 466 385 SE 066 321

AUTHOR Kieren, Thomas

TITLE Language Use in Embodied Action and Interaction in Knowing Fractions.

PUB DATE 1999-10-00

NOTE 20p.; In: Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (21st, Cuernavaca, Morelos, Mexico, October 23-26, 1999); see ED 433 998. Some figures may not reproduce well.

PUB TYPE Reports - Research (143) -- Speeches/Meeting Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.

DESCRIPTORS Arithmetic; Cognitive Structures; Elementary Education; Epistemology; *Fractions; Higher Education; Knowledge Representation; Learning Theories; *Mathematics Education; *Semiotics

ABSTRACT

This essay is based on observation and interpretation of the mathematical language of children while they engage in situations involving fractional numbers. This interpretation includes the consideration of various levels of language use, informal metamorphic and metonymic uses of fractional number language, and the interplay between language use and the children's mathematical construct use. (Contains 10 references.) (DDR)

Reproductions supplied by EDRS are the best that can be made from the original document.

ED 466 385

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

D. Owens

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☐ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

LANGUAGE USE IN EMBODIED ACTION AND INTERACTION IN KNOWING FRACTIONS

Thomas Kieren
University of Alberta
tom.kieren@ualberta.ca

If one takes an embodied view of language use in mathematics knowing then one needs to consider not only the mathematical sign and its referent, but also the person using the language in mathematical knowing actions. But this person's knowing coemerges with a cospecified environment and in particular, with others in that environment. The individual can be viewed as being involved in a number of simultaneous conversations which he or she shapes through action and language use and in which her or his language use is occasioned. This essay is based on the observation and interpretation of use of mathematical language of children, primarily eight years old, as they engage in situations involving the use of fractional numbers. This interpretation includes the consideration of various levels of language use, particularly informal metaphoric and metonymic uses of fractional number language, of the interplay between language use and the children's mathematical construct use, of the roles of interaction in and with environmental features as well as the relationship of language use and mathematical understanding as a process.

Bertrand Russell, in his *Introduction to mathematical philosophy* (1924) suggests that a natural number like three can be characterized as the set of all sets which can be put into one to one correspondence with any constructed trio. It might be expected then that when a person, particularly a child uses the word "three", he or she will be pointing to some exemplary trio. The characteristic of "threeness" is necessarily abstract and relational in character and in making reference to a situation using a sign for three, the child at least unconsciously might be making a one to one correspondence. Of course, we know that young children say, "I am three" without necessarily invoking such a correspondence, although one most often observes such a statement accompanied by the raising of a trio of fingers which is at least a cultural, if not a mathematical correspondence. Even this last rather simple action language example we observe some of the ideas discussed by Sierpinska (1998) in her sociohistorical analysis of mathematical language and communication: children use [mathematical] language to communicate with others [and othernesses], and the child's lines of thought and language use interact.

The discussion above suggests that in using symbols for three in action, a child will not be simply pairing the number word/symbol with a

SE046321

BEST COPY AVAILABLE

referent object in some way, but will be enacting a relationship involving correspondences, or successors for example, intertwining language use with her or his thought. If this is the case for natural numbers like three, it is even more evident for fractional numbers such as three-fourths. Even in the case of one half where it is evident that children have some form of this concept and can use the word half referring to an amount or an active process ("I'm three and a half", "My half is bigger", "I will cut the cake in half") at an early age, using one-half as a word is not simply a process of matching a word to an indicated object., For consider the half piece in Figure 1 below. If one centres that piece on the unit piece vertically many persons, even adults, will notice that the piece no longer "looks like" one half. Thus in using fractional number language, a child will be intertwining that language use with thought/actions which are relational in character and involve what have been observed as constructive mechanisms in children's fractional numbers (Behr et al., 1992; Kieren, 1976; Confrey, 1998; Steffe, 1999) such as partitioning, splitting, or unit reconfiguring and which involve the use of at least proto-ratio notions.

Embodied use of fractional language

The best way to understand the embodied use of fractional language by children is to consider some language samples drawn from the artifacts of such actions. The first set of such examples are given in Figure 2 below. They have been drawn from the work of eight year old children from three different classroom studies of fractional knowing in a suburban school in Edmonton, Canada. These classes involved children with a wide variety of histories of performance in mathematics. Children were involved in using a variety of materials in lessons which were aimed at engaging them with the use of the various fraction sub-constructs - e.g. quotients, operators, measures - (Kieren, 1976). The "fractional pieces" relating to the students' language samples are shown in Figure 1.

These particular fractional language samples were drawn from work done in a lesson which was early in a series of lessons and projects which involved what Simmt (1999) calls variable entry prompts using the first kit in Figure 1 above. They are called prompts rather than problems because it is the students and their lived histories that determine what are taken to be the problems in the setting for each of them and not the set task itself. Their variable entry nature reflects the proscriptive rather than prescriptive nature of the prompts and is evident in the variety of "good enough" responses based on very varied student backgrounds. These prompts and the subsequent student actions and language use were at the heart of lessons which Schonfield (1998) would characterize as emergent - that is the na-

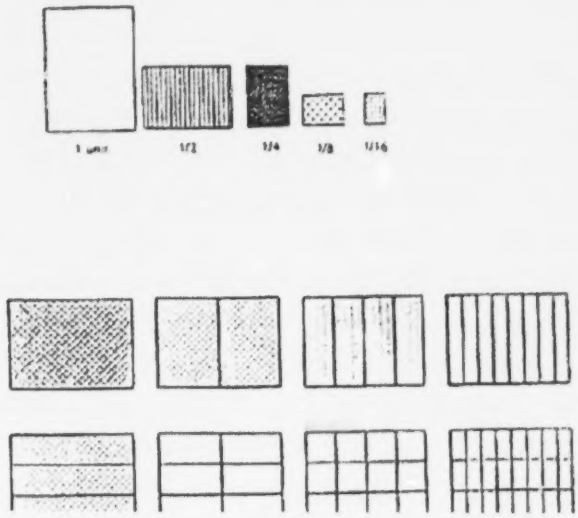


Figure 1. Pieces from Two Fraction Kits

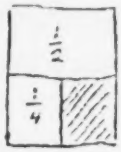
ture and direction of the lesson coemerged from the actions and language and interaction of the students and the subsequent actions and interactions of the teacher; the lesson "direction" was not pre-determined by the teacher, even though the prompt was based on the teacher's view of prior student actions, thinking and language use. In a way the curriculum in this lesson was "wrapped around" the previous lived curriculum of the students and the teacher. In this setting, even though the teacher used standard symbols and words in her communications, the students were free to think about fractions and use fractional language in their own way as long as they could share and explain their work to others. For research purposes, the regular teacher, a research teacher, an observing researcher and a research assistant video taping were present. The language samples below are drawn from student written work or from captured board work, while the interpretation is supported in part by various other "data" sources including tape viewing, transcripts, and research team meeting notes developed daily

Interpretations: How are these eight year olds using fractional number language? In a sense, these diverse examples answer that question on their own. They need no further interpretation. This is even more true if one considers that these samples represent only a part of the observed diversity. Nonetheless, it is useful to address the "title" question of this essay

Here are some things which children wrote or drew about three fourths (3/4).

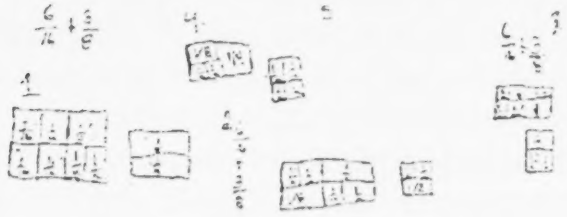
-Three fourths is less than one whole.

-1/2 + 1/4 equals 3/4.



Your turn: On this sheet or on the back write down or draw pictures of 5 things about three fourths.

Here are some of the responses of the children in the class:

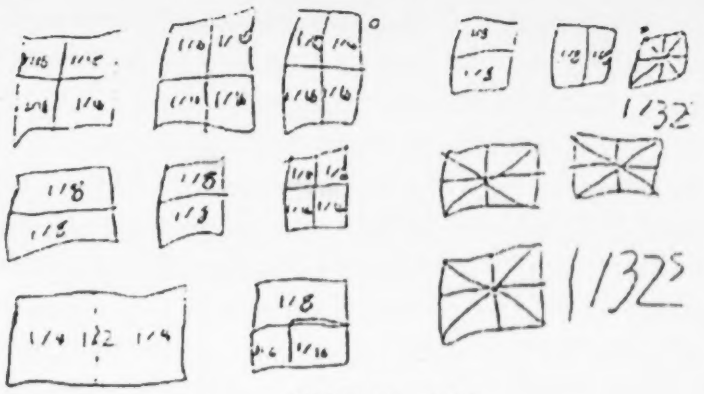


Pat's idea of 3/4

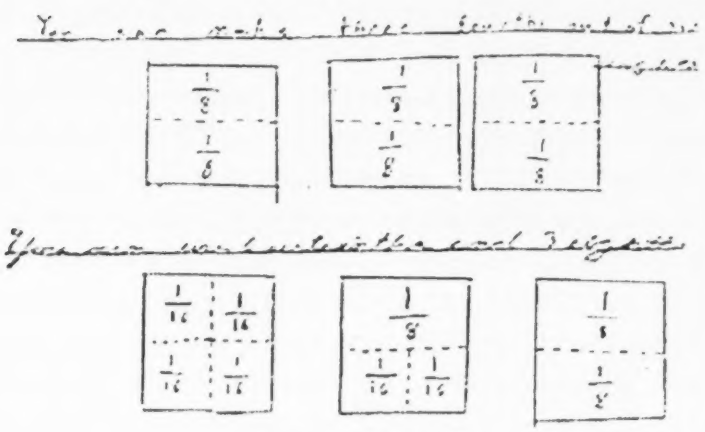
Ricky's idea of 3/4

Figure 2a: Student language use relating to three fourths

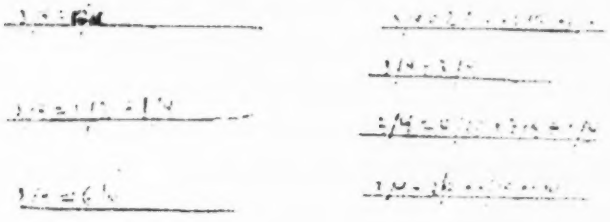
BEST COPY AVAILABLE



Wally's idea of 3/4.



Mark's idea of 3/4

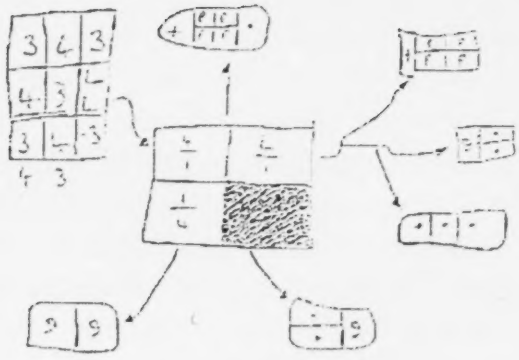


Tad's idea of 3/4.

Figure 2b. Student language use relating to three

BEST COPY AVAILABLE

Sandy's idea of 3/4



Brent's idea of 3/4

Figure 2c. Student language use relating to three fourths

and ask what might it mean to say this language was embodied. For me, the idea of embodied knowing is derived from the work of Maturana and Varela (1987). They see knowing not as representing a pre-given world or even as problem solving per se; instead they see personal embodied knowing as the bringing forth of a world of significance (in our case, involving fractional number mathematics) with others in a sphere of behavioural pos-

BEST COPY AVAILABLE

sibilities. This knowing in action is fully determined by the structure or lived history of the individual; but it is co-specified by the environment which occasions the action (including others in it like the teacher and physical features such as fraction kits and prompts). Looked at another way, the student is observed to act with the energy rich material in the classroom (tasks, materials, comments, and actions by teacher and peers) but in that action is observed to transform that rich input for her or his own use (after vonFoerster,1981). Looking at any of the samples in Figure 2, we can in one way or another observe these actions and transformations. But what of the use of language in all of this? Maturana claims that human beings exist in a world of language and engage in languaging (and perhaps especially in mathematical settings). They can, of course, engage simply in linguistic action or in the consensual coordination of actions. But it is only when human beings recursively coordinate those linguistic actions, or reflect on such actions by making distinctions in them and re-presentations of those distinctions for themselves that humans engage in what Maturana calls languaging. And it is in making such distinctions that humans make things things. Here we observe eight year olds making fractional numbers things for themselves in various ways. They are using fractional number symbols of various sorts to bring forth a world of significance involving fractions. Of course, they are "completing" a task set by the teacher. But had you been there, you would have seen in every one of these classes that the students persisted in working on the task long past its "completion" of saying five things about three-fourths. You would have, for example, seen Jodi and Rachel lying on the floor under their desks writing madly and challenging the class with "We have nine things!" and later "We now have found 23 things [about three fourths]". As might be expected, such claims as well as the work displayed around the classroom both provoked and occasioned other students to continued mathematical action.

A world of significance including fractional numbers Even the limited display in Figure 2 provides a view of the fractional number nature of this "world" and the language used in it. If one imagines that the classroom through the language artifacts of each students' own work as well as the extensive displays and explanations of others work, then one can see how students are offering fractional ideas to others including the teacher and how students might be occasioned to act in this world as well based on reacting to and transforming (for themselves) such artifacts, offerings and explanations. Thus one is aware of how this communication and potential interaction can be observed to expand the cognitive domain of possibilities for the children and even the adults in this setting. Consider for the moment the whole set of work in Figure 2 as well as work from this class in

later figures; one notices the use of a variety of mathematical language - signs for fractions, plus and equal signs. How is this language used? For example, the plus sign is used to indicate the pulling together of fractional pieces to form three-fourths and does not stand for the addition of fractions in the usual sense. But even in these actions, the students are exemplifying the kind of combining and reconfiguring mechanisms envisioned in the work of Behr et al (1992). Thus, an observing researcher or teacher might ask how students might use or transform these actions into more abstract and more formal uses of this language. As these students both create and observe the creation of extensive sets of combinations of fractional units which yield three-fourths, they are generating a background of lived experience for at least the protoconcept of fractional quantities in an equivalence class - one based on a "is as much as" relation. Notice the work of Tad in Figure 2 and especially his sentence 3/4 = 3/4. He, Sandy, and a girl in the class named Jodi (whose other extensive work is not displayed here) were the only students to initially use such sentences. While using a sentence like "1/2 + 1/8 + 2/16 = 3/4" was OK, Jodi for example wasn't sure about using "3/4 = 3/4" even though she said that she knew it was true. Perhaps this query was an indicator of Jodi's awareness of a new use for "=" not as a signal for the result of a computation which would be a very typical reaction of an elementary school child, but as a sign indicating an equivalence relation.

What might researchers or teachers make of the kind of action and language use indicated in Figure 2? They might look to see in what ways the language used pointed to or was intertwined with the kinds of thinking that children were doing. For example, they might ask in what ways does the language used relate to the constructive mechanisms (such as combining or reconfiguring fractional quantities) it might be inferred that students were using? Or they might see the collected artifacts as a source for future knowing actions and lessons. For example, the teacher might use this collection of fractional sentences to occasion children's thinking about equivalence. Researchers or teachers might also ask in what ways the language used indicates similarities and differences in the thinking of students and how actions, utterances or language artifacts such those in Figure 2 illustrate different "levels" of sophistication in language use.

Northrop Frye (e.g. Frye, 1981) has pointed to four levels of historical use of language: hieroglyphic use or language used only in the presence of the object or action; hieratic use which is metamorphic or story-telling in nature in which the language is put for the thought/action with objects; metonymic use in which language use is independent of but analogical to actions. In such a metonymic use the language used can be observed in a

"part for all" relationship with the total action on objects from which it might have originally derived its meaning. Finally in a demotic or analytic use language is independent of any other actions. Without doing too much violence to Frye's ideas, we can use this "taxonomy" to consider the sophistication of fractional language use by children. If we simply stick to the limited samples offered in Figure 2, we can observe several of these "levels" even among the work of these young students. For example, we might note that Pat simply reported on the various reconfigurations of the same set of kit pieces. One could imagine that for him 6/16 + 3/8 was not put for a representation about such action but was directly associated with it and not independent of it at all. We notice in Pat's work, as well as the work of many other students the close tie between fractional language use and the unit fractions associated with the pieces. It might be thought that this somehow indicates the primitive nature of unit fractions for these children, but as we will see in later examples, such thinking and language use is not independent of the setting or of the inferred mechanisms students use in thinking about and acting on fractions. One might think that the work of Richy or Mark or Wally reflects the same kind of hieroglyphic fractional language use as Pat's; but in Wally's case, we see that he is rather excited by invoking the fractional unit of 1/32. Here we see this language use pointing to the Piagetian constructive phenomenon of a child both constructing a new fractional unit from a given one (thirty- seconds from fourths) and using fourths shown as thirty-seconds (or 1/32 s as Wally puts it) to construct another quantity. Thus, it is likely that Wally is using his labeled figures at least to report on or tell stories about his actions on objects. But as the last two examples point to (and subsequent observation of Wally confirm) the labeled drawings have a generative character as well. Notice the adjectival use of language in Mark's samples, especially in the second example where the 6 and 3 (numerators) are adjectives for the sixteenths and eighths. This adjectival use of "numerator" words was observed as typical of many students' early work on tasks with the fraction kit as was the adjectival use of fraction symbols such as 5 1/8 which the children used for 5/8.

Tad's sentences (only a few of which are given) point to a more sophisticated use of fractional language. It is in this work as well as that of a few other students in each of the three experimental classes that we get a sense of a child trying to find [a set of] fractional unit combinations which are quantitatively equivalent to three-fourths. Such language use might be an example of an analogical, part for whole, metonymic use. This is much more clearly seen in the case of Sandy, the last example in the set.

But before turning to a more extensive interpretation of Sandy's work, let us briefly look at Brent's work. Brent could be observed in class to be trying to work like his partners and other children in class and in some way to coordinate his actions with his perception of their actions. He would move pieces around and write down symbols and make drawings that he interpreted to be like those of other children. But an observing teacher, researcher, or even fellow eight year old would find his language use to reflect that fact that Brent's actions were inappropriate to the task; in the language of proscriptive logic, while the other students' actions and intertwined language use were "good enough", Brent's was not. While Brent used numerals and symbols which resembled those of his peers neither his actions nor the semantics or syntax of his language use indicated that he was acting in a fractional number world. Neither Brent's actions nor his symbol use pointed to fractional ideas. Even when his symbols took a fractional form, Brent seemed unaware of the order of the numerals or the meaning of that order. Perhaps this action and language use points out that Brent's structure, his lived history of mathematical activity, simply did not allow him to take appropriate action even in this relatively simple fractional task and environment. For Brent, the language and action of fractions may have been out of his world.

Sandy's actions and related language use indeed show him bringing forth and living in a world of significance including fractional numbers. He used his table generatively to generate all of the possible combinations of a set of fractions which yield three-fourths. To understand his language use better, one needs to understand its history (Kieren and Simmt, 1999). On the previous day, Sandy and his classmates were trying to generate combinations of "half fractions" which were quantitatively equivalent to five-fourths. Sandy claiming that he was "trying to find all of them", had made up a chart with similar headings to those of his in Figure 2. He was observed standing at the board, looking up in the air, and occasionally making an entry in his table, now written on the board, reflecting his dreamed up combination that worked. During this time, he was interrupted by a teacher and other students who wanted him to explain what he was doing. Frustrated by this interruption, he started to work secretly on a new five-fourths table which he did not finish, but in which instead of using the table to record thought up combinations he used it to generate an orderly sequence of such combinations. Sandy's work in Figure 2 then represents his reluctant response to the teacher's request to "make a table like you did yesterday". But now when he presented it to her after only a minute or two, he confidently said "There, that's all of them!". Sandy's use of his table now had become formalized and was an analytic means of dealing

with fractional number combinations which was independent of any actual combining of quantities in any physical sense or even of imagining such combinations one by one. In his work then unlike any of the other work in Figure 2, we see fractional language used not with action, not to report action, nor even as an analogy to action, but as an independent thought action in itself. Such language use might be considered an example of demotic or analytic language use.

Several things should be noted about this "leveled" way of looking at fractional language use. These levels are not to be construed as stages of language use such that once a child reaches a certain stage, then her or his language use will continue to reflect that achievement. Neither is language use abstract and context free. Whatever the use of language, it occurs in and coemerges with the context in which it occurs and in the conversations with others within it. Both of these issues will be revisited in examples which follow. Finally, while it is beyond the scope of this essay to elaborate on this, language use in action is one of the useful indicators in the observation of a person's growing and changing mathematical understanding viewed as a process (e.g., Pirie and Kieren, 1994)

Changing use of fractional language

The fractional language samples in Figure 2 above represent a variety of language uses by different students in the face of a particular prompt. To get further insight into the nature of fractional language in embodied action, it is interesting to look at such language use by one child across tasks (and across time). The samples in Figure 3 below come from the work of Kara considered by her teacher to be a "typical Grade Three" student.

The first sample offered in Figure 3 arose as Kara and four partners worked on the following task: Each student in the group was to fold 4 unit sheets, one into halves, one into fourths, one into eighths and one into sixteenths. They were then asked to shade in and identify a fractional part of each. They then put all of their "fractions" into a common set and worked together to find relationships among twenty elements in that set. Before turning to Kara's writing, it is interesting to note that well over ninety percent of the students in the Grade 3 classes studied were able to predict the number of parts given the number of folds - at least from one to the next. All seemed very aware of the multiplicative and even exponential nature of this typical splitting task confirming the observations that Jere Confrey has made on this over the years.

There are many interesting features of Kara's use of fractional language. One striking feature of this is the use of "words" instead of fractional symbols. This occurred even though the teacher and some other students used fractions to describe folded and shaded amounts. A second

4 eights = one half
4 sixteens = one quarter
1 half of a half = a quarter
1 half of a half of a half = a half of a quarter
4 eights = 1 half
2 fourths = 1 half
4 sixteens and two eights = 1 half
1 half = 1 half
8 sixteens = 1 half
1 fourth one eighth and two sixteenths = 1 half

Figure 3. Kara's use of fractional language

BEST COPY AVAILABLE

feature that Kara shared with nearly all of her class mates is evidenced in her use of "eights" and "sixteens". At first, the teacher was convinced that the children simply were not hearing the "th" at the end of the fraction words. But since this use was almost universal in this class and persisted even when the teacher emphasized the " th", the language use might be attributed to what the student perceived in the situation. As noted in the last two items in the first sample of Kara's work, fractions and fractioning in this setting were related to a process of [to an observer] iterative folding. The resulting parts occurred not as discrete items, but as connected sets. Thus, three halvings resulted in an "eights" set.

The second set of Kara's fraction language samples was drawn from very early in her experience with "half fraction kit" (Figure 1). Notice that her previous language use persisted, but changed as well. For Kara, it was only gradually that the fractional pieces (and related numbers) became units for her which could be combined and reconfigured and used in their own right as suggested by Behr et al (1992).

The third sample in Figure 3 is drawn from Kara's work on the task related to Figure 2 above. Notice that while complex fractional symbols were used, they on describe or report on previously completed actions on the physical materials where the unit fractions are used as labels. One might say that Kara was using fractional language metaphorically or using it to report on prior action. Notice also that the fraction symbols are not combined with other mathematical signs, even though Kara appeared to be thinking in terms of equivalences- as much as. Her drawings show that her fractional combinations were compared with (and in her drawings placed on) a unit or one. Thus, it appears that Kara's fractional language use changed with the setting of its use (and of course was related to but not dictated by the fractional language uses of her peers and the teacher). Further, it appeared that this language uses was becoming more "mathematical" and perhaps sophisticated with time and experience.

The fourth and last sample of Kara's work in Figure 3 allows us to comment on the last conjecture, but before turning to it, consider the two language samples in Figure 4 below. Barney, the boy who generated those samples was considered by his teacher to be mathematically gifted. The first of his samples arose as he worked on a task which was a student favorite - the Missing Fraction Mysteries. In the particular case, the students were to describe a "hidden" fractional amount from the two clues that it was more than one fourth and less than three fourths. From observing the work of his friend Sam, Barney noted that one could find such an amount "subtractively" by deleting a fractional amount from one. But more than that, he realized that one could engage in fraction play, adding and sub-

tracting amounts which finally "solved" the mystery. The first sample in Figure 4 was just one of a full page of various such mathematically playful solutions which were created in their own terms (a demotic/analytic use) independent of and unrelated to any "concrete" actions.

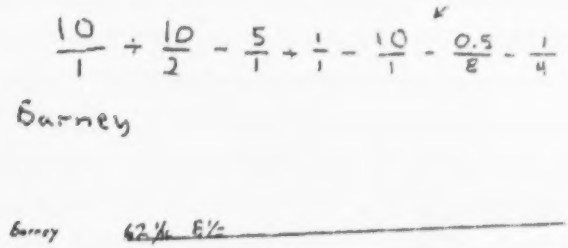


Figure 4. Barney's use of fraction language

It might be thought that Barney had very sophisticated mathematical and symbolic control over at least a subset of the fractional numbers which he could invoke in any setting involving fractions. But his second sample allows one to question this assumption. Notice that Barney had reverted to using an adjectival form of fractional language where 62 1/16 was used to describe the length of a long table measured on a "fractional measures scavenger hunt" using a constructed fraction tape which was a unit (one metre) folded into sixteenths. The second fraction, 8 1/2, is Barney's measure of the same table using a "half tape". Notice here that Barney's use of language is no longer abstract, but descriptive. Although later on another measuring task Barney noted that one could just measure with any tape and convert that measure to a measure based on any other fraction tape, at least early in his experience with fractions as measures Barney did not think in those terms. In Pirie and Kieren's terms (1994), Barney had folded back to a more local mode of understanding and perhaps his language use reflects this.

Turning back now to Kara's last fractional language sample in Figure 3, we see her reports of two measures of the same long table. Notice that although Kara had previously used fractional language in a more sophisticated manner with fraction kit pieces or fractions as discrete units, in this new "fraction as measure" setting her language looks much less sophisticated.

BEST COPY AVAILABLE

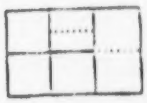
By looking at student fractional language use in various settings, a researcher or teacher needs to be aware that the students perceive those settings in their own terms and their fractional language use reflects their own structures, the nature of that setting and their lived experience in it. Such an embodied view of fractional language use suggests that while the taxonomy of levels is a useful observation tool, embodied fractional language use is not a monotonically staged phenomenon.

Fractional language use in interaction

The interpretations of the fractional language samples discussed so far have emphasized two things. Students use fractional language in a constructive manner - that is, their language use is related to and intertwined with their thinking and their re-presentations in fractional settings. Secondly, the context of the language use matters. The various samples in the figures above suggest that fractional thinking and language use, even if it is determined by the structure of the individual child, is coemergent with the setting in which it occurs. In this final interpretive section we turn to the communicative uses of fractional language. Figure 5 below contains two items which were drawn from one urban, multi-ethnic middle school class of 11 and 12 year olds with very varying histories of mathematics achievement. As a "warm-up" task during the introduction to fractional numbers, the teacher had asked the class to find the amount represented by 1/6 + 3/12 + 2/24 and to express this amount as a fraction. In doing this task students had available a fraction kit like the second one in Figure 1 above. Because of the very wide variety of "solutions" the teacher had many of the students sketch and explain their solutions. Van's work is typical of many of the solutions and explanations which involved the kit. Peter, a boy who considered himself to be a good student, apparently wanted to generate an "answer" which was different from any of the others. He went to the board and started the interchange which occurs in the second part of Figure 5 below.

Here we see Peter use fractional language in many ways. First he uses "eights" as a means of being different. While his quickly generated solution is correct, it was done mainly to "be clever". After being challenged Peter (3) uses fractional language as part of or intertwined with his thinking in which he can be observed to recombine units (trying to think of how eighths could be related to sixths, twelfths and twenty-fourths). Finally because he wants his thinking and fractional language use to be accepted in this conversation he shows how the language he used could be seen as analagous to the fractional action of inter-relating the eighths units and the sixths, twelfths and twenty-fourths which many of his peers had worked

Van: It makes $\frac{6}{12}$.



Justification:

Change $\frac{2}{24}$ into $\frac{1}{12}$, and change $\frac{1}{6}$ into $\frac{2}{12}$.

1. **Peter:** Use eights. Four eights. (Making this drawing)

2. **Teacher :** Wow! How did you ever think of that? (emphasis mine)

– – – brief pause – – –

3. **Peter:** Half of a twelfth plus one twenty-fourth plus one fourth of a sixth is an eighth – and that happens twice.
   Half of a twelfth plus half of a twelfth plus a fourth of a sixth is an eighth – and that happens twice
   Altogether there are four eighths.

4. **Teacher (Like nearly all of the class and the observer):** What?

5. **Peter:** See: I'll show you. (Makes this drawing)

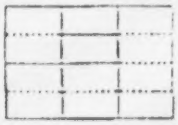


Figure 5. Fraction use in interaction

BEST COPY AVAILABLE

with. In this action we observe Peter using fractional language in many ways to communicate things about himself and his thinking to others and to deliberately address himself to his peers in what he thinks might be their terms.

Concluding remarks

There are many ways to think about fractional language and its use. One could do this abstractly by thinking of the various ways in which fractional symbols might be related to situations to which they refer. Such an analysis would have one ask " how might fractions be used in setting x and why might this be so?" Or one might treat fractions in a textbook fashion where one would focus on how children might come to write, relate and symbolically combine fractions in ways which match some pre-given, prescribed outcomes. Such an analysis might prompt one to ask "how well do children perform with fractions and how might such performance be enhanced?" Rather than taking either of the approaches above, I have chosen to focus on the use of fractional language as an embodied phenomenon. In so doing I have tried to show how even young children use fractional language to think about, act in and bring forth a world which includes fractional mathematics. The interpretations offered here focused both on how the children used the symbols and on how this symbol use and related thinking could be observed to be occasioned by the environment and community in which they existed. Thus fractional language use was observed as subject to the internal structural dynamics of the individual and the social/ interactional dynamics of the community in which he or she existed all-at-once. Further to the extent that the children were observed to be acting in mathematically legitimate ways with fractions such actions and language use were also embodied in the broader culture of the practices of mathematics.

What difference does such an interpretive approach make? First I think allows researchers and teachers to understand the necessary diversity in a class which is coming to know fractions and use them in action. Such knowing and related use of fractional language appears to occur at many levels of sophistication both across the children in a classroom but also within the knowing actions of any one child. But such knowing is clearly related to the context and setting in which it occurs. Based on such observations the teacher can wrap the curriculum around the thinking, actions and language use of the children while still acting to expand their potential domain and sophistication of fractional language use. Such an interpretation does allow one to think about the relationship of fractional symbol to referential context through the context sensitive embodiment of the fractional lan-

guage user. In addition it prompts one to think about curriculum and teaching of fractional number ideas so as to take into account the ways in which students use fractional language intertwined with their thinking, which includes the constructive mechanisms of children's mathematics; and so as to promote fractional language use as part of the maturing use of mathematics in a person's life with others.

References

Behr, M., Harel,G., Post,T. and Lesh, R. (1992), Rational Number, ratio, and proportion. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.296-233). New York: MacMillan Publishing Co.

Confrey, J. (1998). Building mathematical structure within a conjecture driven teaching experiment on splitting. In Berenson, S.B., Dawkins, K.R., Blanton, M., Coulombe, W.N., Kolb, J., Norwood, K. and Stiff, L. (Eds.) *Proceedings of the twentieth annual meeting of the North American Chapter of the International group for the Psychology of Mathematics Education.* Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. pp.39-50.

Frye, N. (1981). *The great code.* Toronto: Academic Press.

Kieren, T. and Simmt, E. (1999). *Some notes on recursion as a tool for observing mathematics knowing in action.* A paper given at the 1999 AERA Annual Meeting. Montreal, April.

Pirie, S. and Kieren, T. (1994). Beyond Metaphor: formalising in mathematical understanding within constructivist environments. *For the learning of mathematics,* 14(1) 39-43.

Russell, B. (1924) *Introduction to mathematical philosophy.* London: Allen and Unwin.

Schonfield, A.H. (1998). On theory and models: The case of teaching in context. In Berenson, S.B., Dawkins, K.R., Blanton, M., Coulombe, W.N., Kolb, J., Norwood, K. and Siff, L. (Eds). *Proceedings of the twentieth annual meeting of the North American Chapter of the International group for the Psychology of Mathematics Education.* Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. pp. 27-39.

Sierpinska, A., (1998). Three epistemologies, three views of classroom communication: constructivism, sociocultural approaches, interactionism. . In H. Steinbring, M. Bartonlini Bussi and A. Sierpinska (Eds.) *Language and communication in the mathematics classroom.* Reston VA: National Council of Teachers of Mathematics. pp. 30-62.

Steffe, L. (1999). Personal communication.
von Foerster, H. (1981). *Observing systems*. Seaside, CA: Intersystems Publications.

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

NOTICE

Reproduction Basis

☑ This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

☐ This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").